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ANISOTROPIC TRANSVERSE FLOW AND THE HBT CORRELATION FUNCTION

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ABSTRACT

Utilizing the Lorentz invariance of the correlation function we study the effects of anisotropic transverse flow on the HBT correlation function. In particular we show that directed flow would evidence itself by non zero "side–long" and "side–out" terms in the correlation function. We also show that the study of ratios of the correlation functions evaluated on different event subsamples and/or different Lorentz systems provides information on the source dimensions and velocity which could be less biased by corrections of different kinds. We present a fitting technique appropriate for HBT analysis which does not require binning and is especially useful for multidimensional fitting.

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1 Introduction

The Hanbury-Brown–Twiss (HBT) analysis of multiparticle production processes is becoming a widely used technique. It provides information on the space-time evolution of an excited strongly interacting system produced in high energy collisions. In recent years many experiments have collected high statistics data which permits detailed multidimensional analysis of the correlation function. From the theoretical point of view many valuable results have been obtained, both in the understanding the of the HBT method itself and in the ways it can be applied to the data [1]–[5]. It is now clear, for example, that the correlation function depends sensitively on the dynamics of the expansion of the system. Most of the theoretical observations have been made using different kinds of models to describe this expansion.

The discovery of transverse directed flow in the collisions of ultrarelativistic nuclei [6] implies that the HBT analysis of such data should take the effects of flow into account. In our previous publication [7] we used RQMD generated events to show how directed flow affects the extracted HBT radii. Utilization of the Lorentz invariance of the correlation function, defined as the ratio of the invariant two particle distribution to the product of two invariant one particle distributions, can provide valuable information about the source and can be used as a check of the validity of the assumptions contained in certain models. These questions are addressed in the current paper mostly in the context of directed flow. In particular we argue that directed flow would evidence itself by non zero "side–long" and "side–out" cross terms in the correlation function, even if the correlation function is determined with an azimuthally symmetric event sample (case in which the reaction plane is not determined).

In dealing with the experimental data we find that the conventional method of fitting for the correlation function (especially multidimensional fitting) has several disadvantages and sometimes gives biased results, in spite of many improvements and suggestions made recently [8]–[9]. The main problems relate to the low statistics in some bins and to a proper evaluation of fluctuations in the distributions of mixed

pairs. Multidimensional fitting is essential for the questions under consideration, and we present a fitting method without binning which is free from the problems mentioned above and is very flexible in its applications.

The paper is organized in a following way. We start with Lorentz properties of the correlation function and discuss how the extracted source radii depend on the rapidity of the frame in which the analysis is done, and how anisotropic transverse flow affects the correlation function. We then discuss the ratio of the correlation functions evaluated with different event subsamples and/or in different Lorentz systems and discuss how these ratios could help in the selection of the appropriate functional form for the correlation function and in the extraction of parameters of the source such as the anisotropic flow velocity. Finally we present the new method of fitting the correlation function which does not require binning.

2 The HBT correlation function in different frames

The objective of an HBT study of the space-time evolution of multiparticle production is typically the determination of the effective source dimensions, time duration of particle emission and the velocity of the source. Boson interferometry is a very effective tool for this purpose, but like any other tool has its own limitations. The first limitation, a rather trivial one, is that one can study only that part of the source which emits particles into the experimental acceptance. This very simple statement mathematically can be written as space-momentum correlation of the source function. It implies that the study of the correlations between particles produced in different parts of phase space permits one to measure the source size as it seen from different directions.

The second limitation is not so trivial. It arises from the physics of interferometry. *Interferometry measures the distance between pions at the instant the second pion is produced.* This distance depends not only on the size of the source, but also the duration of the emission (which affects the distance traversed by the first pion before the second

pion is produced) and the velocity of the source (the source could move to another place before the production of the second pion). Mathematically it is related to the fact that only three of the four components of the pair momentum difference q_i are independent, because of the particles being on the mass shell. The fourth component of the momentum difference can be expressed through first three components and the pair velocity $\mathbf{V}$:

$$q_0 = \mathbf{V}\mathbf{q}. \tag{1}$$

Often this results in the situation when there are more unknowns than the number of parameters possible to extract experimentally, and additional assumptions must be made about the source to resolve the problem.

The correlation function, being a ratio of invariant distributions, is invariant with respect to a Lorentz transformation:

$$C(\mathbf{q},\mathbf{P}) = \frac{d^6n/d^3p_1d^3p_2}{d^3n/d^3p_1d^3n/d^3p_2} = C(\mathbf{q}',\mathbf{P}'), \tag{2}$$

where $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ is the total momentum of the pion pair, and the prime denotes the values in any other Lorentz system. Lorentz invariance provides us with the possibility to calculate the correlation function in any frame provided we know its functional form and value in any other frame. Below we use the following notation: by an asterisk (*) we denote the values in the source rest frame (but we use y^* for the source rapidity in the laboratory frame), and a tilde ($\tilde{}$) is used for the LCMS (Longitudinally Co-Moving System) frame, in which the longitudinal velocity of the pair is equal to zero ($\tilde{V}_z = 0$).

For example, using this notation, the Lorentz transformations between the analysis frame (with rapidity y) and the source rest frames reads:

$$q_z^* = q_z \cosh(y - y^*) + q_0 \sinh(y - y*), \tag{3}$$

$$q_0^* = q_0 \cosh(y - y^*) + q_z \sinh(y - y*). \tag{4}$$

The longitudinal and transverse velocity components of the pair in the analysis frame are:

$$V_z = \tanh(\tilde{y} - y), \tag{5}$$

$$V_\perp = \tilde{V}_\perp / \cosh(\tilde{y} - y). \tag{6}$$

Very often the correlation function is written in the form:

$$C(q) = 1 + \lambda e^{-\mathcal{Q}(q)}. \tag{7}$$

Using the small q approximation in the source rest frame (we discuss below the existence of such a frame) one can represent $\mathcal{Q}$ as:

$$\mathcal{Q} = a_0 q_0^{*2} + a_x q_x^{*2} + a_y q_y^{*2} + a_z q_z^{*2}. \tag{8}$$

Our notation differs here from that often found in the literature ($a_0 = \tau^2$; $a_x = R_x^2$; $a_y = R_y^2$; $a_z = R_z^2$). We have introduced the new notation since the fits to experimental data can give negative values for these parameters, especially if there are cross-terms. Throughout this paper we use the coordinate system in which the z axis is along the beam direction, the x axis is along the "out" [10] direction (the direction in the transverse plane along the momentum of the pair) and the y axis is along the "side" direction, which is perpendicular to the "out" and beam directions. Note that in the source rest frame the corresponding cross terms $a_{x0} = a_{y0} = a_{z0} = 0$, if the source velocity is defined as [7]:

$$v_i = \frac{\langle r_i t\rangle - \langle r_i\rangle\langle t\rangle}{\langle t^2\rangle - \langle t\rangle^2} = 0. \tag{9}$$

We also *assume* that the source is azimuthally symmetric (in the source rest frame): $a_{xy} = a_{yz} = 0$, $a_x = a_y = a_\perp$, and there is no correlation of the "x-z" type: $a_{xz} = 0$.

As was mentioned above only three components of q_i are independent, thus the experimental data can be fitted by the correlation function in the form:

$$\mathcal{Q} = A_x q_x^2 + A_y q_y^2 + A_z q_z^2 + A_{xz} q_x q_z. \tag{10}$$

The correspondence of our parameters A to the often used R^2 is obvious, and we have adopted again a new notation for the reason given above. We assume that the source can move only longitudinally (we consider the more general case in the following

section) and make use of the Lorentz invariance of the correlation function. After some algebraic manipulations one finds the relations between the parameters A, which are to be extracted from the data analyzed in the analysis frame defined by rapidity y, and the source parameters a_i introduced above:

$$
\begin{aligned}
A_z &= \frac{1}{\cosh^2(\tilde{y}-y)}[a_z \cosh^2(\tilde{y}-y^*) + a_0 \sinh^2(\tilde{y}-y^*)] && (11)\\
A_x &= a_\perp + \frac{\tilde{V_\perp}^2}{\cosh^2(\tilde{y}-y)}[a_0 \cosh^2(y-y^*) + a_z \sinh^2(y-y^*)] && (12)\\
A_y &= a_\perp && (13)\\
A_{xz} &= \frac{2\tilde{V_\perp}^2}{\cosh^2(\tilde{y}-y)}[a_z \cosh(\tilde{y}-y^*)\sinh(y-y^*) \\
&\quad + a_0 \cosh(y-y^*)\sinh(\tilde{y}-y^*)]. && (14)
\end{aligned}
$$

In the case when the analysis frame coincides with the source rest frame ($y = y^*$) these equations (11–14) give the well known equalities:

$$
\begin{aligned}
A_z &= a_z + a_0 V_z^2, && (15)\\
A_x &= a_\perp + a_0 \tilde{V_\perp}^2, && (16)\\
A_y &= a_y, && (17)\\
A_{xz} &= 2a_0 \tilde{V_\perp} V_z. && (18)
\end{aligned}
$$

When the analysis frame coincides with the LCMS ($y = \tilde{y}$) we have:

$$
\begin{aligned}
A_z &= a_z \cosh^2(\tilde{y}-y^*) + a_0 \sinh^2(\tilde{y}-y^*), && (19)\\
A_{xz} &= 2\tilde{V_\perp} \sinh(\tilde{y}-y^*)[a_z - a_0], && (20)\\
A_x - A_y &= \tilde{V_\perp}^2 [a_0 \cosh^2(\tilde{y}-y^*) + a_z \sinh^2(\tilde{y}-y^*)], && (21)
\end{aligned}
$$

and it follows:

$$
\begin{aligned}
A_z - \frac{A_x - A_y}{\tilde{V_\perp}^2} &= a_z - a_0, && (22)\\
\frac{A_{xz}/\tilde{V_\perp}}{A_z + (A_x - A_y)/\tilde{V_\perp}^2} &= \tanh(2(\tilde{y}-y)). && (23)
\end{aligned}
$$

The last equation, in principle, permits to evaluate the velocity of the source with respect to the pair velocity.

The equations presented above can be used as a check of the validity of the functional form (Gaussian) used for the correlation function. One needs to evaluate A_i as a function of analysis frame rapidity y and compare the observed dependence of the parameters on the rapidity with the dependence given by these expressions. One of the simplest tests of this type would be to look for the independence of the parameter A_y (often called the "side radius squared") on the analysis frame rapidity y.

In the discussion above it is assumed that the source rest frame exists, that is that all pions entering the detector are emitted by a source with fixed rapidity. In general this is not true. It is rather likely that the effective source should be described as a superposition of sources moving with different rapidities. In this case, approximating the correlation function by a single (multidimensional) Gaussian will not be successful; it could result, for example, in the dependence of A_y, the effective source "side" size, (and/or the coherence parameter λ) on the analysis frame rapidity. This arises from the fact that the radii A_i (other than A_y) depend on the analysis frame rapidity y (see Eq.14). If the source parameter a_y is different for the different sources, a fit to a single Gaussian results in an apparent y-dependence for the A_y. Note that in the saddle point approximation [5] the effective pion source is exactly in the (Gaussian) form considered above. In this case the question about the existence of the source rest frame is equivalent to a question about the validity of the saddle point approximation.

3 Directed flow effects

Here we discuss how directed flow affects the terms in the correlation function related to the "side" direction. In particular we argue that in the presence of directed flow the "side–long" and "side–out" cross terms of the correlation function become non zero. The experimental measurements of these terms (especially as a function of centrality

of the collision) would be of great interest.

We start with the simple case when the correlation function can be written in the "source rest frame" in a pure Gaussian form (8–10) without any cross terms except the trivial ones related to the fact that particles are on the mass shell. We assume also that directed flow exhibits itself simply as a movement of the source in the transverse plane. Under this assumption one can calculate the correlation function in the analysis frame using Lorentz transformations. To do this we start with the expression for the correlation function in the source rest frame(8–10), then perform a Lorentz shift in the direction of the transverse flow (with transverse rapidity $y_\perp$), and then do a Lorentz shift along the z axis to the frame moving with rapidity y. In the final step we use the mass shell constraint, Eq. (1). We denote by ψ the angle between the direction of the source velocity (the reaction plane) and the x axis ("out" direction).

This gives the expressions:

$$
\begin{aligned}
A_z &= \frac{1}{\cosh^2(\tilde{y}-y)}(a_z\cosh^2(\tilde{y}-y^*) + a_0\sinh^2(\tilde{y}-y^*)\cosh^2 y_\perp \\
&\qquad + a_\perp\sinh^2(\tilde{y}-y^*)\sinh^2 y_\perp), \qquad (24)\\
A_x &= a_\perp\sin^2\psi + a_\perp(\cos\psi\cosh y_\perp + \frac{\tilde{V_\perp}\cosh(y-y^*)\sinh y_\perp}{\cosh(\tilde{y}-y)})^2 + \\
&\quad + a_0(\frac{\tilde{V_\perp}\cosh(y-y^*)\cosh y_\perp}{\cosh(\tilde{y}-y)} + \cos\psi\sinh y_\perp)^2 + a_z\frac{\tilde{V_\perp}^2\sinh^2(y-y^*)}{\cosh^2(\tilde{y}-y)}, \qquad (25)\\
A_y &= a_\perp(\cos^2\psi + \sin^2\psi\cosh^2 y_\perp) + a_0\sin^2\psi\sinh^2 y_\perp, \qquad (26)\\
A_{xy} &= 2a_\perp\sin\psi\cosh y_\perp(\cos\psi\cosh y_\perp + \frac{\tilde{V_\perp}\cosh(y-y^*)\sinh y_\perp}{\cosh(\tilde{y}-y)}) \\
&\quad + 2a_0\sin\psi\sinh y_\perp(\frac{\tilde{V_\perp}\cosh(y-y^*)\cosh y_\perp}{\cosh(\tilde{y}-y)} + \cos\psi\sinh y_\perp) \\
&\quad - a_\perp\sin(2\psi), \qquad (27)\\
A_{xz} &= 2a_\perp\sinh y_\perp\frac{\sinh(\tilde{y}-y^*)}{\cosh(\tilde{y}-y)}(\cos\psi\cosh y_\perp + \frac{\tilde{V_\perp}\cosh(y-y^*)\sinh y_\perp}{\cosh(\tilde{y}-y)}) \\
&\quad + 2a_0\cosh y_\perp\frac{\sinh(\tilde{y}-y^*)}{\cosh(\tilde{y}-y)}(\cos\psi\sinh y_\perp + \frac{\tilde{V_\perp}\cosh(y-y^*)\cosh y_\perp}{\cosh(\tilde{y}-y)})
\end{aligned}
$$

$$+2a_z\tilde{V_\perp}\sinh(y-y^*)\frac{\cosh(\tilde{y}-y^*)}{\cosh^2(\tilde{y}-y)}, \tag{28}$$

$$A_{yz} = (a_\perp + a_0)\frac{\sinh(2y_\perp)\sin\psi\sinh(\tilde{y}-y^*)}{\sinh(\tilde{y}-y)}. \tag{29}$$

Let us consider, for example, the "side–long" cross term A_{yz}. Averaging it directly over the reaction plane angle ψ gives zero, but the expression one has to average is not A_{yz}, but the correlation function $\exp(-A_{yz}q_yq_z)$. Then directed flow effectively results in nonzero cross terms in the correlation function. Note that analogous terms appear also in A_x, A_{xy}, A_{xz}.

If no selection of events is done with respect to the reaction plane orientation, averaging over ψ gives:

$$\frac{1}{2\pi}\int d\psi\exp(-A_{yz}q_yq_z) = I_0(B_{yz}q_yq_z), \tag{30}$$

where

$$B_{yz} = (a_\perp + a_0)\frac{\sinh(2y_\perp)\sinh(\tilde{y}-y^*)}{\sinh(\tilde{y}-y)}, \tag{31}$$

and one could fit the data using the corresponding form of the correlation function.

We should mention that strictly speaking the averaging over the flow angle should be performed separately in the numerator and denominator in the definition of the correlation function, Eq. (2), rather than averaging the correlation function itself. The above result is valid if the width of the two particle correlation related just to the movement of the source (this correlation has nothing to do with the quantum interference of the identical particles) is much wider than the width of the correlation due to quantum interference. This problem/assumption is relevant not only to anisotropic flow effects. One should not forget that the HBT correlations are always "on top" of any long and/or short range correlations due to resonance decays, source movement, energy and momentum conservation, etc.. By fitting the correlation function in a form appropriate only for quantum interference correlation we implicitly assume that all other correlations have much greater widths.

Formulae (24–29) can also be useful for purposes not directly related to transverse flow. For example, the case of $\psi = 0$ and $\tanh(y_\perp) = V_\perp \cosh(\tilde{y} - y^*)$ would give the parameters of the correlation function as measured in the pion pair rest system.

4 The ratio of the correlation functions

We now suggest a method to study the functional form of the correlation function, exploiting its Lorentz transformation invariance. First we hypothesize a particular functional form of the correlation function. We then calculate the parameters in different Lorentz systems and compare the observed dependence on the rapidity of the analysis frame with the expected dependence. In other words we choose the form and parameters of the correlation function and test for its Lorentz invariance.

In practice we suggest the fitting of the ratio of the correlation functions:

$$R(a_x, a_y, a_z, a_0; \lambda, v_x, v_z) \equiv \frac{dN^{(1)}_{true}}{dN^{(2)}_{true}}, \tag{32}$$

where (1) and (2) denote different event subsamples and/or different Lorentz systems. Note that in this method there is no need to produce mixed pairs; it also much less affected by the corrections of various types (such as the Coulomb correction).

To illustrate the method we consider an example related to directed flow. One can assume that the source of pions in events with different reaction plane orientation differs only in orientation of directed flow velocity, but all other parameters describing the source are the same. Then one can perform a fit to extract the source transverse velocity. One should fit the ratio:

$$R(a_x, ...) = \frac{1 + \lambda e^{-\mathcal{Q}'}}{1 + \lambda e^{-\mathcal{Q}}}, \tag{33}$$

where $\mathcal{Q}'$ and $\mathcal{Q}$ differ only by a Lorentz shift in the flow direction. For example, if one can select two event subsamples with the flow direction pointing along the $+x$ direction and the $-x$ direction, then $\mathcal{Q}'$ and $\mathcal{Q}$ would be defined by formulae (24–29) with $\cos\psi$ equal to $+1$ and -1, respectively.

One could argue that the source might look different from the $+x$ and $-x$ direction [7], that is the parameters of the effective sources emitting pions in these directions are different. In this case it could be better to use event subsamples corresponding to flow directed along the $+y$ and $-y$ directions. In this case, because of symmetry, the only difference between $\mathcal{Q}'$ and $\mathcal{Q}$ is due to the direction of the movement of the source.

Note that a simple test of whether the parameters of the source are the same for different event subsamples (selected, for example, according to the orientation of the pair momentum with respect to the reaction plane) could be the evaluation of the correlation function using invariant variables. The simplest case would be a one dimensional fit in Q_{inv}. If the parameters of the source for different event subsamples are the same, and the only difference between them is the direction of the motion of the source, the extracted R_{inv} should be the same for all event subsamples, due to the Lorentz invariance of the correlation function.

5 Likelihood function fitting

The usual approach for the fit of the correlation function assumes the introduction of binning in some part or even over the entire phase space. This approach has a number of disadvantages. It is sensitive to the experimental acceptance and to cuts introduced during the analysis. Often some bins have a low number of hits, which introduces biases in the results. Sometimes the parameterized correlation function has only a few parameters, but the complicated functional form of the parameterization may demand binning in many dimensions.

It is attractive in this case to use the maximum likelihood fitting technique, which is free from almost all of the problems mentioned above. However a direct application of this technique is difficult, because in this approach one needs to parameterize not only the correlation function itself (the goal of the fit), but of the distributions of "true" and "mixed" pairs. The latter is extremely difficult, since one must take into account

numerous nontrivial acceptance and analysis cuts.

We propose a method based on the likelihood function, which allows one to avoid these problems. For simplicity we consider one dimensional case, but the generalization of the method for multiple dimensions is straightforward. In the one dimensional case the correlation function is written as:

$$C(x) \propto (\frac{dN_{true}}{dx})/(\frac{dN_{mixed}}{dx}). \tag{34}$$

Let us denote the probability density for mixed pairs as $p(x)$:

$$\frac{1}{N_{mixed}}\frac{dN_{mixed}}{dx} = p(x) \tag{35}$$

Then the probability density for the true pair distribution can be written as

$$\frac{1}{N_{true}}\frac{dN_{true}}{dx} = G[C]C(x)p(x), \tag{36}$$

where G is the normalization factor, which is a functional of $C(x)$, and therefore depends on the parameters which determine C.

The log-likelihood function depends on the correlation function in the following way:

$$\ln L[C] = \sum_{k=\{true\ pairs\}} \{\ln G[C] + \ln C(x_k) + \ln p(x_k)\}. \tag{37}$$

The last term in the sum does not depend on $C(x)$ and can be omitted. But one must evaluate the normalization factor $G[C]$ for different sets of parameters. The problem is that for this calculation one needs to know the probability density $p(x)$. The trick of our method is that the required integral can be evaluated using “mixed” pair distribution.

$$G^{-1}[C] = \int dx C(x)p(x) = \int dx C(x)\frac{1}{N_{mixed}}\frac{dN_{mixed}}{dx} \tag{38}$$

$$\approx \frac{1}{N_{mixed}} \sum_{i=\{mixed\ pairs\}} C(x_i) \tag{39}$$

For example, if we consider the Gaussian form of the correlation function

$$C(x) = 1 + \lambda \exp(-ax^2), \tag{40}$$

the log-likelihood function can be written as

$$\ln L(a,\lambda) = \sum_{k=\{true\ pairs\}} \ln(1+\lambda\exp(-ax_k^2)) \tag{41}$$

$$-N_{true}\ln\{\frac{1}{N_{mixed}}\sum_{i=\{mixed\ pairs\}}(1+\lambda\exp(-ax_i^2))\}. \tag{42}$$

The method has many advantages. First, it gives a correct evaluation of the errors in the fit, because it automatically takes into account the fact that the pairs in the mixed event technique are not independent and in general the fluctuations are proportional to $\delta N_{mixed} \propto N_{mixed}^{3/4}$ [11], in contrast to the fluctuations in true pairs distributions $\delta N_{true} \propto N_{true}^{1/2}$. Taking into account that the maximum number of mixed pairs which can be generated is proportional to the square of the number of events ($N_{mixed} \propto N_{events}^2 \propto N_{true}^2$), it follows that the fluctuations in mixed pairs distributions *never* become much less than that in the real pairs distribution, regardless of how many of them are generated. Other advantages of the likelihood technique is that it is free from the problems associated with finite bin size, and it is very flexible in its applications (For example, it is easy to introduce analysis or acceptance cuts, and it is very straightforward to use different functional forms of the correlation function.)

6 Conclusion

We have presented new techniques for the analysis of like-particle pairs using the HBT method and for testing the form of the correlation function used in such studies. Starting from the Lorentz-invariant property of the correlation function, we derive relationship for the fitted parameters as a function of the rapidity of the analysis frame. This technique is particularly useful, but not limited to, studies of interacting systems in which anisotropic transverse flow is present.

Our results may be summarized as follows: (i) The analysis of the same data in different Lorentz systems helps in the extraction of the source parameters and assessing the validity of the approximations used, for example the validity of Gaussian source

(saddle point approximation). (ii) Anisotropic transverse flow would result in the non zero "side" cross terms in the correlation function. (iii) The study of ratios of the correlation functions evaluated in different Lorentz frames (the ratios of "true" pair distributions) is an important new analysis tool. (iv) We have presented a new fitting technique based on the likelihood function which does not require binning and has many advantages in comparison to the conventional method.

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